JAN 15 1964

N64-15782

CODE-1

CR-53067

15p.

Technical Report No. 32-441

Fuel-Containment Requirements for Gaseous-Fuel Nuclear Rockets

Robert V. Meghreblian

OTS PRICE

XEROX $ 1.10 ph.

MICROFILM $ 0.80 mf.

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

September 2, 1963

Technical Report No. 32-441

Fuel-Containment Requirements for Gaseous-Fuel Nuclear Rockets

Robert V. Meghreblian

Robert V. Meghreblian

Robert V. Meghreblian, Chief
Space Sciences Division

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

September 2, 1963

Prepared Under Contract No. NAS 7-100
National Aeronautics & Space Administration

CONTENTS

TABLES

FIGURES

15782

ABSTRACT

author

Fuel-containment requirements for open-cycle gaseous-fuel nuclear rockets are examined for systems in which there is physical contact between propellant and fuel. Such systems necessarily allow some loss of nuclear material. Analysis shows that the total fuel lost during the propulsion period is the prime constraint in determining containment requirements. A parameter, the containment factor, is introduced to provide a measure of fuel-containment efficiency. Application to representative high-thrust (booster) and low-thrust (interplanetary) vehicles is considered. The analysis indicates that containment factors fifty times as large as those presently demonstrated experimentally are required, in order to limit nuclear fuel loss during propulsion to the order of 1000 kg.

AUTHOR

I. INTRODUCTION

Several open-cycle gaseous-fuel nuclear reactors of the type required for rocket propulsion have been proposed which involve the use of either fluid-dynamic or fluid-magnetic phenomena for preferentially trapping the nuclear material in the gaseous cavities of the reactor (Ref. 1). Whatever the trapping mechanism, the essential requirement is that the fuel be trapped, or contained, in gas phase while the gaseous propellant either flows through (in a diffusion process) or around the fuel. In systems utilizing diffusion (Ref. 2), the fission energy is transferred from the nuclear material to the propellant by atomic interaction, whereas in systems with separate fuel and propellant regions, the energy transfer is accomplished principally by thermal radiation from fuel to propellant (Ref. 3 and 4). In all these situations the fuel and propellant are in physical contact, and it is to be expected that some of the nuclear material will be swept out of the cavities by the propellant into the exhaust nozzle and thereby lost to the system.

In addition to the apparent biological hazard arising from this discharge of nuclear material into the outer environment, consideration must be given to the question of the degradation in engine performance due to the presence of a high-molecular-weight species in the exhaust and the question of economics imposed by the high cost of nuclear fuel. Each of these considerations imposes separate constraints on the containment requirements.

The purpose of this Report is to examine the implications of these constraints, and to show how they are related to the reactor and rocket engine characteristics, and how these in turn ultimately influence overall vehicle parameters.

II. CONTAINMENT FACTOR

For the purposes of this analysis it is convenient to speak in terms of a containment factor ψ, which is defined as

$\psi \equiv$ ratio of average density of fuel to propellant in reactor cavities divided by the ratio of the average densities in the rocket exhaust

Thus the quantity ψ gives a direct measure of the excellence of the fuel-containment method; the larger this factor, the more effective the containment mechanism. If ρ_{FC} and ρ_{PC} denote, respectively, the average densities of nuclear fuel and propellant in the cavities, and ρ_{FRC} and ρ_{PRC}, those in the rocket chamber (and exhaust), then

$$\psi = \frac{\rho_{FC}/\rho_{PC}}{\rho_{FRC}/\rho_{PRC}} \tag{1}$$

It is implied in this formulation that the fuel-propellant mixture leaving the reactor cavities passes into a rocket motor chamber and thence out the exhaust nozzle. Thus whatever fuel leaves the cavities is lost to the system.

Another quantity of interest is the average concentration ratio of nuclear material to propellant in the rocket exhaust n_{FP}:

$$n_{FP} \equiv \frac{<N_{FRC}>}{<N_{PRC}>} \tag{2}$$

where $<N_{PRC}>$ is the concentration of the propellant and $<N_{FRC}>$ the concentration of the fuel in the rocket chamber. On the basis of rocket motor performance characteristics alone, it is possible to determine a maximum allowable value for n_{FP}. This constraint arises because of the great disparity in the molecular weights of nuclear fuels and efficient propellants. For plutonium and molecular hydrogen this ratio is approximately 120; thus in a nuclear rocket engine of given power, the addition of one part of plutonium to 120 parts of pure hydrogen would double the average molecular weight of the exhaust mixture and reduce the specific impulse by about 20%. In order that no appreciable degradation in specific impulse be sustained, it is necessary that the fuel-propellant concentration ratio be kept small. It is required, therefore, that $n_{FP}A_F/A_P \ll 1$, where A_i is the molecular weight of species i. For plutonium-hydrogen mixtures, this requirement yields

$$n_{FP} \leq 10^{-3}$$

The upper limit 10^{-3} is no constraint at all and results in the loss of a total mass of nuclear fuel per propulsion period which is 12% of the mass of propellant ejected, a very large amount indeed. Obviously, the principal constraint on the containment factor will be due to the total allowable mass of fuel which may be expended.

To establish the relationship between the containment factor ψ and the total fuel expended, a relationship is first required between ψ and n_{FP}. This is easily established by writing the quantity ψ in terms of the average concentrations:

$$\psi = \frac{<N_{FC}>}{<N_{PC}> n_{FP}} \tag{3}$$

where $<N_{FC}>$ and $<N_{PC}>$ denote the average concentration of fuel and propellant, respectively, in the cavities. The quantity n_{FP} in turn is related to the total weight of nuclear fuel expended per propulsion period W_F^{ex} through the expression

$$n_{FP} = \frac{W_F^{ex}}{W_P}\left(\frac{A_P}{A_F}\right) \tag{4}$$

with W_P denoting the total weight of propellant discharged during this same period. Thus all that is required now is the fuel-to-propellant mass ratio. This is easily established, given a suitable criterion. Since it has already been observed that the total mass of fuel expended is the critical consideration, it is convenient to introduce this criterion in terms of the cost of the fuel. For simplicity, this is expressed in terms of relative cost of the vehicle. That is to say, the criterion for allowable fuel loss is selected on the basis of the fraction of total vehicle cost to be invested in nuclear fuel. Thus if the criterion selected is the requirement that the fuel expenditure equal the value of the rest of the vehicle (less payload), then the fuel-to-propellant weight ratio may be expressed as

$$\frac{W_F^{ex}}{W_P} = \frac{p_P}{p_F}\left[1 + s\frac{p_T}{p_P} + \frac{p_N}{p_P}\left(\frac{a}{\lambda a_e}\right)\right] \tag{5}$$

where p_i is the cost per pound of component i, λ is the propellant-to-vehicle gross weight ratio, a_e is the rocket engine thrust-to-weight ratio, a is the vehicle acceleration at start of the propulsion period, and s is the tank-to-propellant weight ratio; the subscript T denotes tank and N reactor. (In the examples which follow, the following values are used: $p_F/p_P = 40{,}000$, $p_N/p_P = 800$, $p_T/p_P = 400$, $p_F = 10{,}000$ dollars/lb and $s = 0.05$.)

III. INFLUENCE OF SOLID-FISSION FRACTION ON CONTAINMENT

An indication of typical values of the containment factor and of the corresponding nuclear fuel lost per propulsion period is obtained from two examples. In the first, we consider the containment required for a single-stage booster to place a 100,000-lb satellite in Earth orbit. In the second, we consider a low-thrust interplanetary vehicle for transferring a 400,000-lb payload from Earth-satellite orbit to a circular orbit about Mars (at 1.1 Mars radius). These two cases illustrate the two basically different propulsion applications possible with gaseous-fuel reactors.

The comparison of containment requirements on the basis of a common mission objective is a convenient measure of the relative investment in nuclear material necessary to achieve improved vehicle characteristics (i.e., increased payload fraction). Previous analyses have shown that the performance of a gaseous reactor as a rocket engine could be controlled by varying the distribution of fissionable material between a conventional mode (e.g., solid-fuel plates) and the gas cavities (Ref. 1). This distribution was specified in terms of the solid-fission fraction f, the fraction of the total reactor fission power released in the solid phase. It is convenient to introduce this parameter as the independent variable in the present analysis. A major objective, of course, is to diminish f to zero in order to achieve the highest specific-impulse ratio possible. This requires, however, relatively large concentrations of nuclear material in gas phase and therefore large containment factors. Thus, in the final analysis, the efficiency of the containment process determines the maximum fraction of the total fuel mass that can be retained in gas phase and, consequently, vehicle performance. Now, it may very well be that containment factors achievable in practice will be much too small to allow substantial increases in specific impulse over that possible with the "equivalent Rover" reactor (Ref. 5). In that event, gas-phase fuel containment will, of course, be uninteresting. The examples which follow indicate that the choice between a high- and a low-thrust application has some bearing on this question.

IV. HIGH-THRUST APPLICATION

The satellite mission reported in Ref. 6 is selected for this example. The characteristics of two different single-stage vehicles to perform this mission are summarized in Fig. 9 and 10 of that paper. In the present calculation, we consider only the case $\beta = 10^{-3}$ [refer to Eq. (6), Ref. 6], which represents a gaseous reactor in which the gaseous mixture of fuel-propellant is quite transparent, having emissivities in the order of 10^{-3}–10^{-2}. From the engine and vehicle characteristics given in Ref. 6, the quantities ψ and W_F^{ex} can be directly computed by means of Eq. (3) and (5). The results are plotted in Fig. 1 as a function of the solid-fission fraction. The corresponding value of the specific-impulse ratio ($I \equiv I_c/I_s$, where I_i denotes the specific impulse corresponding to the temperature T_i) is also shown for convenient reference. The principal trend to note is that as the amount of fuel in gas phase is increased (i.e., as $f \rightarrow 0$) so as to increase the specific impulse, the containment requirement becomes more severe. This behavior reflects two effects: the higher concentration of nuclear material in gas phase and the smaller total mass (and therefore cost) of the vehicle. Clearly, as more of the fuel is retained in solid form, the containment requirement is rapidly reduced; unfortunately, so also is the performance.

The choice of the quantity $<N_{PC}>\psi$ for summarizing the results in Fig. 1 stems from the fact that the average propellant concentration (i.e., cavity pressure level) is somewhat arbitrary and determined by other considerations. In order to obtain explicit values for ψ, a value for $<N_{PC}>$ must be selected. Table 1 summarizes the results for the present example on the premise that $<N_{PC}>$ is three times the total particle concentration $<N_{RC}>$ in the rocket chamber, with

$$<N_{RC}> = \frac{p_{RC}}{kT_c} \tag{6}$$

The symbol p_{RC} denotes the pressure in the rocket chamber (here taken as 100 atm), k is the Boltzmann constant, and T_c, the temperature to which the propellant is heated

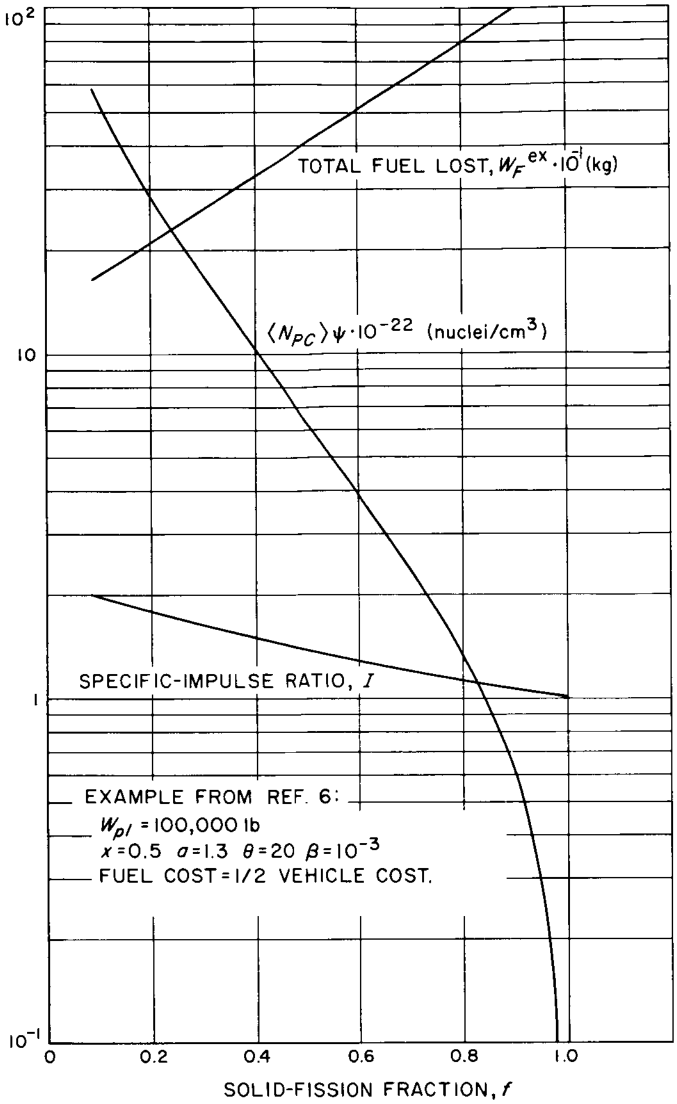

Fig. 1. Fuel-containment requirements for single-stage satellite booster (V = 1.14)

in passing through the reactor cavities. The temperature T_c corresponds to the specific impulse I_c (achieved by gas-phase heating), and in this analysis T_c and I_c are assumed to be related by the function shown in Fig. 2. The relationship for $T_c \leq 20{,}000$°K is taken from the work of Altman (Ref. 7), and for $T_c > 20{,}000$°K, the perfect gas relation was used assuming complete dissociation and ionization.

On the basis of the parameters selected the required values of ψ increase from about unity for the low-

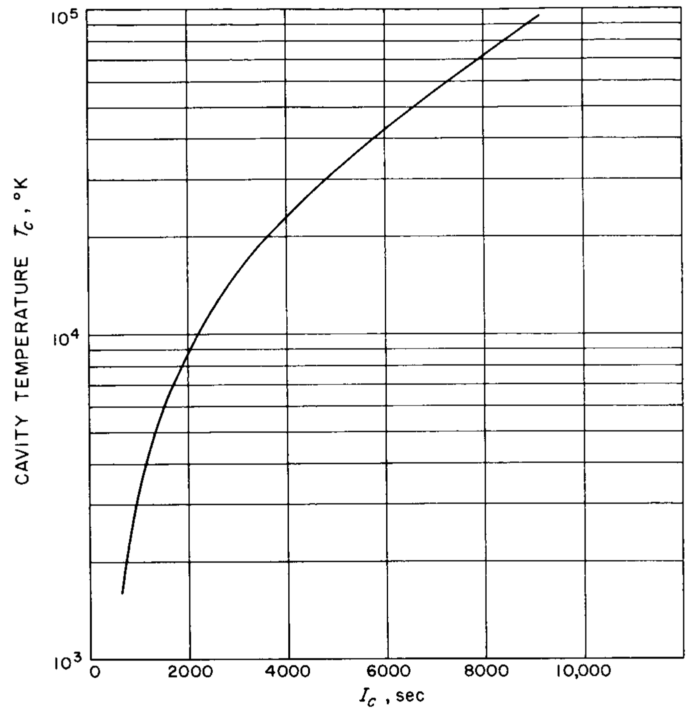

Fig. 2. Temperature–specific impulse relationship for hydrogen

Table 1. Single-stage satellite booster (V = 1.14)

f	$\langle N_{FC} \rangle$ (nuclei/cm^3) $\times 10^{-16}$	$\langle N_{FC} \rangle \psi$ (nuclei/cm^3) $\times 10^{-22}$	I_c sec	T_c °K	$\langle N_{PC} \rangle$ (nuclei/cm^3) $\times 10^{-20}$	ψ	ρ_{FC}/ρ_{PC}
0.090	730	58.25	1400	5400	4.03	1443	2.17
0.165	450	34.7	1300	4700	4.63	750	1.166
0.29	252	18.5	1130	3950	5.51	336	0.549
0.5	97	6.39	950	3100	7.02	91	0.166
0.91	9	0.5325	730	2050	10.62	5.01	0.0102
0.98	1.73	0.09975	700	2000	10.9	0.915	0.00191

Parameters (Ref. 6):

W_{pl} = 100,000 lb $x = 0.5$ $a = 0.3$ $\theta = 20$

$\beta = 10^{-3}$ fuel cost = ½ vehicle cost p_{RC} = 100 atm performance systems (i.e., f $\simeq$ 1) to values of over a thousand for systems with considerable gas-phase heating. It is of interest to note that actual containment experiments using the vortex method (Ref. 2) have yielded to date values of $\psi \simeq 1.1$. The results of Table 1 indicate that regardless of the choice of $<N_{RC}>$, values of ψ in the order of at least ten must be attained to allow as much as a 10% increase in the specific impulse over that possible with the equivalent Rover engine. Even then, the attendant loss in nuclear fuel is in the order of 1000 kg per propulsion period.

The last column of Table 1 lists the average density ratio of fuel to propellant in the reactor cavities. This quantity is of some interest since it corresponds to the $\bar{w}$ used in the earlier work of Ref. 2.

Inasmuch as the criterion selected for the containment factor was that the value of the nuclear fuel lost per propulsion period would be equal to one-half the total cost of the vehicle, it is to be expected that ψ will depend upon the vehicle size. This effect is shown in Fig. 3, in which the containment factor is given as a function of the vehicle gross weight, for fixed values of the solid-fission fraction. The corresponding values of the payload to be placed in Earth orbit are shown in the cross plots. As expected, larger vehicle sizes allow poorer containment. It is noted, however, that even at $f = 0.5$ (which yields $I = 1.34$), and a 400,000-lb payload, $<N_{PC}> \psi$ is smaller by only a factor of two than in the corresponding case for a 100,000-lb payload. This results in a 2.65-million-lb vehicle and a loss of about 1800 kg of nuclear material per propulsion period. The dependence of $<N_{PC}> \psi$ on vehicle size arises only through the average

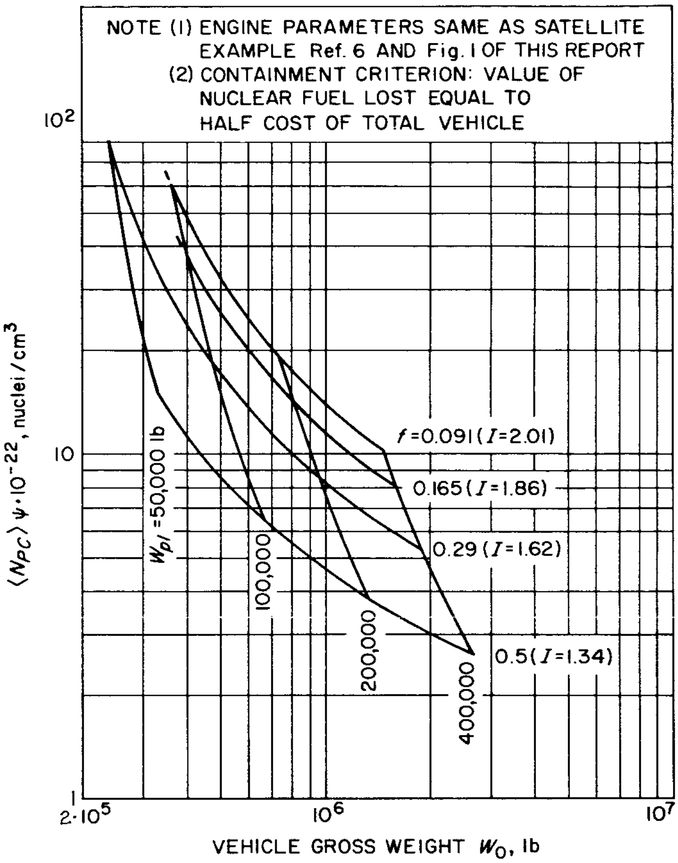

Fig. 3. Influence of vehicle size on containment factor for single-stage satellite booster ($V = 1.14$)

fuel concentration $<N_{FC}>$ and reflects simply the variation in reactor size with system size. As the vehicle size increases, at fixed f, the reactor size increases and the critical fuel concentration decreases.

V. LOW-THRUST APPLICATION

The low-thrust application of gaseous-fuel reactors appears to be most promising for missions to the near planets (Ref. 5). On this basis a Mars-orbiter mission was selected as an interesting representative case. The calculations were carried out on two separate bases. In the first, a minimum-energy trajectory was used for the Earth-Mars transfer with planet-centered spiral escape and capture trajectories at each end. In the second, an "equivalent Mars orbiter" analysis was performed using the burnout velocity equation for a single-stage vehicle in gravity- and drag-free flight. This second calculation was performed to allow direct analytical techniques to be applied in relating and examining vehicle performance and reactor criticality in terms of vehicle characteristics. In both approaches, however, a single set of engine parameters and a single initial acceleration at Earth orbit (10^{-3} g) were selected. Again, as in the high-thrust problem, the solid-fission fraction was taken as the independent variable. In the present calculation, however, the specific impulse was optimized for each value of f. This was possible because in the low-thrust application a radiator is incorporated into the rocket engine complex (Ref. 8). The introduction of a radiator effectively decouples the temperature of the gas mixture in the cavities from the temperature of the engine solids, thereby allowing unlimited increases in specific impulse (at least in principle). Thus the low-thrust application allows an additional degree of freedom in the choice of engine characteristics. The implications in regard to the containment factor in these systems is considered in the discussion that follows.

The engine parameters selected for the minimum-energy Mars trajectory were based on the case used in Ref. 8. These are:

$$\theta = 0.3 \qquad x = 0.3 \qquad \sigma_a^{(FC)} = 1000 \text{ barns}$$
$$\eta = 16 \qquad \zeta = 0.1 \qquad \text{BeO moderator}$$

In Ref. 8, it was assumed that the gas in the reactor cavities was transparent and that there was a linear relationship between temperature and enthalpy. More recent analyses (Ref. 9) indicate, however, that for hydrogen propellant, especially in the lower-performance regime, a more realistic relationship is that the temperature varies as the ⅔ power of the enthalpy. Also, there is a good possibility that a practical fuel-propellant gas mixture might be entirely opaque. On this basis, the ⅔ power law was selected, and it was further assumed that the gas mixture in the cavities was opaque to thermal radiation. In general, for a gaseous reactor system with radiator and an opaque gas in the cavities, the specific-impulse ratio is given by (Ref. 9)

$$I^2 = \frac{1}{\mu}\left[1 + \gamma(1-\mu) - \beta_s\left\{\left[\vartheta(I^{4/3} - 1) + 1\right]^4 - 1\right\}\right] \tag{7}$$

with

$$\beta_s \equiv \frac{\sigma\epsilon_c A_c T_s^4}{\dot{m}h_s} \qquad \vartheta \equiv \frac{T_g - T_s}{T_c - T_s} \qquad \gamma \equiv \frac{P_r}{\dot{m}h_s}$$

and $\mu \equiv f + \zeta(1 - f)$. The remaining symbols are defined as follows: ϵ_c and A_c are the emissivity and radiating area respectively of the gas in the cavities; T_s is the temperature of the engine solid, T_g the effective radiating temperature of the gas mixture and T_c, the maximum internal temperature of the gas mixture; $\dot{m}$ is the mass flow rate of propellant; h_s is the enthalpy/mass of the propellant at T_s; P_r is the power rejected by the radiator; σ is the Stefan-Boltzmann constant; and ζ is the fraction of the fission energy released in the gas phase and not attenuated by the gas mixture.

Equation (7) yields the specific impulse ratio I, given the solid-fission fraction f, the radiator power fraction γ, and the quantities β_s and ϑ. (Typical solutions for this equation are shown in Fig. 5 of Ref. 9.) The engine performance in terms of I is related to the engine weight by introducing certain dimensionless parameters. The appropriate relations have been derived in Ref. 8, and these are applied directly to the present problem. The principal engine components comprising the gaseous nuclear rocket are the reactor, the radiator, the propellant and tanks, and the payload. All other components are ignored in the weight analysis of the overall vehicle.

The optimization of I for a given value of the solid-fission fraction is based on the maximization of the payload fraction π_{pl}, given the initial acceleration a and the set of engine parameters in Eq. (7). From Ref. 8,

$$\pi_{pl}(I, \mu) = 1 - \frac{a}{\theta I}\left[(1 + s)\,\tau(I) + (1 + \eta\theta)\,\gamma(I, \mu) + 1\right] \tag{8}$$

where $\gamma(I, \mu)$ is obtained from Eq. (7), and $\tau(I)$, the dimensionless "burning time," is given by

$$\tau(I) = \frac{\theta I}{a}\left(1 - e^{-V/I}\right) \tag{9}$$

It may be shown that for this system the payload fraction passes through a maximum π_{pl}^{max} at I_*, which is given by the solution to the following equation:

$$1 + (1 + s)\frac{V\theta}{a} e^{-V/I_*}$$

$$- \frac{1 + \eta\theta}{1 - \mu} \left\{ 1 + \beta_s + \mu I_*^2 + \beta_s \left(\vartheta + \frac{13}{3} \vartheta\, I_*^{4/3} - 1 \right) \right.$$

$$\left. \times \left[\vartheta \left(I_*^{4/3} - 1 \right) + 1 \right]^3 \right\} = 0 \qquad (10)$$

Here $V \equiv \Delta v / c_s$, where Δv is the equivalent velocity increment corresponding to minimum-energy transfer ellipse from Earth to Mars using planet-centered spiral escape and capture trajectories at each end, and c_s is the exhaust velocity corresponding to the maximum solid temperature T_s. For the present example, using T_s = 2000°K and hydrogen propellant, $V = 2$ gives a good approximation for the equivalent Mars orbiter. The solution for I_* from Eq. (10) is shown as the broken-line curve in Fig. 4. The solid-line curve is the actual I_* computed for the minimum-energy transfer ellipse. The agreement is seen to be good.

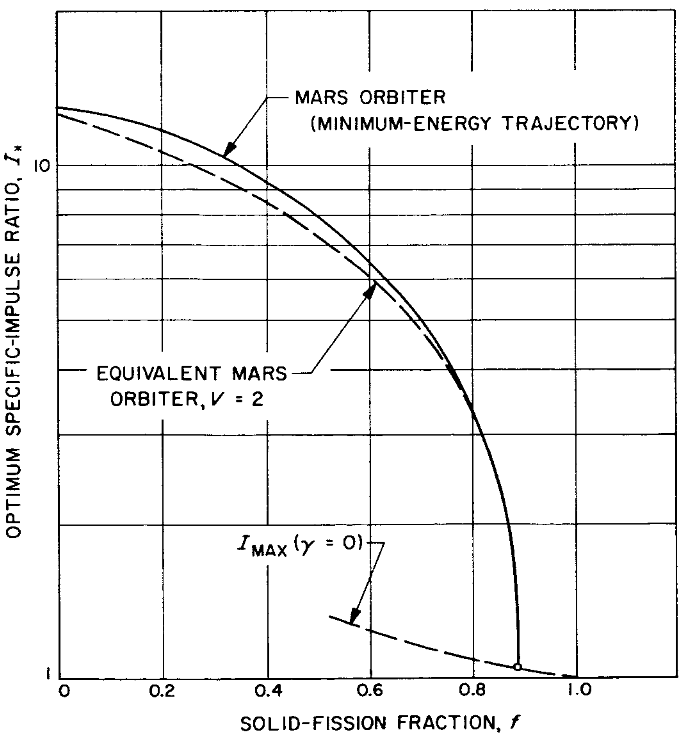

Fig. 4. Optimum specific-impulse ratio as function of solid-fission fraction for Mars orbiter

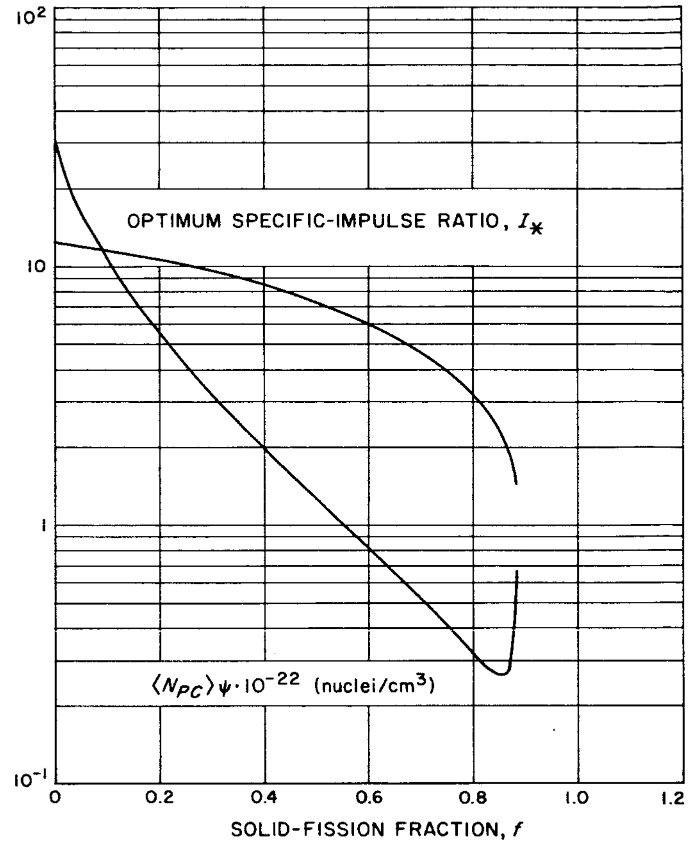

Fig. 5. Fuel-containment requirements for equivalent Mars orbiter (V = 2)

The results for the optimum specific impulse I_* for a given value of the solid-fission fraction may be used in the set of engine and vehicle parameter relations to determine the characteristics of the optimized system. These results are summarized in Table 2 for the case $V = 2$, using the same choice of $<N_{PC}>$ as previously. The symbols at the top of Table 2 are: π_{pl}^{max}, the payload fraction corresponding to I_*; W_0, the vehicle gross weight at start of propulsion period; l, the diameter of a cylindrical reactor of equal length and diameter; and W_X, the weight of the reactor. The containment factor and the optimum specific-impulse ratio are plotted in Fig. 5 as a function of the solid-fission fraction, and the vehicle gross weight and total fuel lost, in Fig. 6.

The most important trend in these results is the increase in the fuel lost as the containment becomes poorer (i.e., as ψ decreases). For example, at the low value of $<N_{PC}> \psi = 2.6 \times 10^{22}$, the corresponding fuel lost during a single propulsion period is about 6000 kg. In order to reduce the fuel lost by an order of magnitude the figures indicate that the containment factor must be increased by a factor of roughly 20. Note that even this

Table 2. Vehicle and engine characteristics for equivalent Mars orbiter ($V = 2$)

f	I_*	γ	π_{pl}^{max}	W_0 lb	l cm	W_N lb	$\langle N_{FC} \rangle$ (nuclei/cm^3) $\times 10^{-18}$	$n_{FP} \times 10^{-4}$	W_F^{ex} kg	$\langle N_{PC} \rangle \psi$ (nuclei/cm^3) $\times 10^{-22}$	T_c °K	$\langle N_{PC} \rangle$ (nuclei/cm^3) $\times 10^{-19}$	ψ	ρ_{FC}/ρ_{PC}
0	12.61	40.44	0.7837	511,000	115.6	5593	22.48	0.750	306	30	89,000	2.45	12,250	110
0.1667	11.02	55.83	0.7274	550,000	137.6	9454	6.978	1.030	509	6.78	66,000	3.29	2000	25.4
0.286	9.771	63.81	0.6790	590,000	153.2	13,020	4.053	1.184	668	3.43	53,000	4.10	837	11.86
0.440	8.039	69.30	0.6017	665,000	174.9	19,380	2.180	1.310	1050	1.665	38,300	5.68	293	4.61
0.60	6.113	68.00	0.4914	815,000	203.6	30,610	1.108	1.350	1640	0.822	25,500	8.52	96.5	1.56
0.74	4.276	57.93	0.3451	1,160,000	244.9	53,230	0.5235	1.213	2860	0.431	15,400	14.1	30.5	0.445
0.80	3.385	48.28	0.2549	1,570,000	275.9	76,150	0.3416	1.078	4100	0.317	11,100	19.6	16.12	0.208
0.85	2.500	33.61	0.1603	2,495,000	316.8	115,200	0.2190	0.834	6220	0.263	7300	29.8	8.83	0.0882
0.86	2.281	29.01	0.1396	2,860,000	326.1	125,600	0.1986	0.750	6804	0.265	6500	33.5	7.91	0.712
0.871	2.025	23.08	0.1191	3,360,000	332.3	133,000	0.1810	0.633	7238	0.286	5500	39.6	7.23	0.0549
0.880	1.622	12.85	0.1007	3,970,000	314.8	113,000	0.1764	0.409	6243	0.432	3900	55.8	7.75	0.038
0.8815	1.440	8.184	0.0997	4,010,000	286.5	82,280	0.1946	0.293	4784	0.665	3400	63.4	10.5	0.0369
0.890	1.050	0	0.103	3,880,000	150.3	12,320	0.664	0.050	895	—	2000	—	—	—

Parameters:

$\theta = 0.3$ $\quad \zeta = 0.1$ $\quad s = 0.05$ $\quad W_{pl} = 400{,}000$ lb

$\eta = 16$ $\quad \vartheta = 0.1$ $\quad \sigma_a^{(FC)} = 1000$ barns $\quad a = 10^{-3}$

$x = 0.3$ $\quad \beta_s = 0.1$ $\quad$ BeO moderator

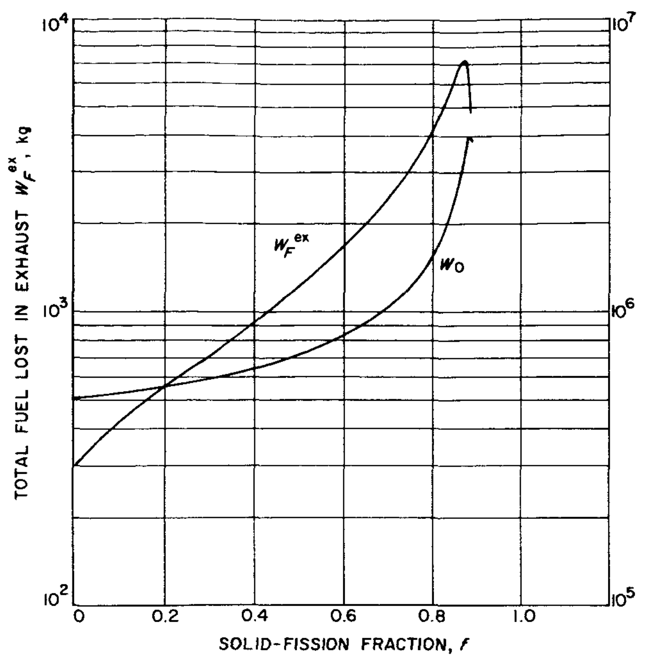

Fig. 6. Fuel lost and vehicle gross weight for equivalent Mars orbiter ($V = 2$)

case, at the present cost of nuclear material, results in a cost per flight of some 12 million dollars for the fuel alone. Along with the increase in fuel lost with decreasing fractions of gas-phase heating in the reactor, there is an increase in vehicle gross weight (see Fig. 6 and Table 2). This trend, which occurs also in the case of the high-thrust systems (such as the satellite booster discussed earlier), reflects directly the degradation in engine specific impulse with increasing f (see Fig. 5). Thus, as more of the fissionable material is contained in solid (or any temperature-limited) form, the performance is reduced until at $f = 1$, the limiting value for I of the equivalent Rover system is reached. This trend in specific impulse is monotonic in f; the effect on the gross weight is also monotonic in the case of the high-thrust systems. In the case of the low-thrust systems, however, a new trend is noted at the high values of f. In the present example of the equivalent Mars orbiter, the gross weight passes through a maximum at $f = 0.881$. This behavior stems from the additional degree of freedom allowed in the engine through the radiator power-fraction parameter [see Eq. (7)]. This influence is apparent in Table 2. It was noted in the earlier discussion that the specific-impulse ratio was optimized for a given value of f. This yields a specific value of γ. At small values of f, and therefore large values of I, the radiator size is large, but the reactor size is small. At large values of f, the optimum specific-impulse ratio is near unity and the corresponding radiator requirement is small, but the reactor size is large.

Thus, as f increases from zero, the total engine weight (radiator plus reactor) at first increases. Beyond a certain point, however, the radiator requirement becomes so small that the total engine weight now decreases. The net result is that the engine thrust-to-weight ratio passes through a minimum and the vehicle gross weight through a maximum.

This behavior may be demonstrated analytically. The problem is to determine the extremum of $\pi_{pl}^{\max}(I_*, \mu)$, given the constraint of Eq. (10), which may be written $\phi(I_*, \mu) = 0$. The application of the method of Lagrangian multipliers yields the relation

$$\frac{\partial \pi_{pl}^{\max}}{\partial I_*} + \lambda_0 \frac{\partial \phi}{\partial I_*} = 0 \tag{11}$$

where λ_0 is the Lagrangian multiplier. But, by definition, the first of these derivatives is zero; therefore, so also must be the second. Thus the extremum of $\pi_{pl}^{\max}$ is obtained by the simultaneous solution of Eq. (10) and $\partial\phi/\partial I_* = 0$. It may be shown that this yields the single equation in I_*:

$$VE(I_*)\,G(I_*) - 2I_*(1+\eta\theta)\left[D(I_*) - G(I_*)\right] - \frac{16}{9}\beta_s\,\vartheta(1+\eta\theta)I_*^{1/3}\,D(I_*)\,C(I_*)\,A^2(I_*) = 0 \tag{12}$$

with

$$A(I) \equiv 1 + \vartheta(I^{4/3} - 1)$$

$$B(I) \equiv \frac{13}{3}\vartheta I^{4/3} + \vartheta - 1$$

$$C(I) \equiv 13\,\vartheta I^{4/3} - \vartheta + 1$$

$$D(I) \equiv I^2\left[E(I) + 1 + \eta\theta\right] + 1$$

$$E(I) \equiv \frac{\theta V}{aI^2}(1+s)\,e^{-V/I}$$

$$G(I) \equiv (1+\eta\theta)\left[1 + \beta_s + I^2 + B(I)A^2(I)\right]$$

The solution to Eq. (12), I_*^m, corresponds to the maximum noted previously. The complete set of parameters consistent with this solution is

$$I_*^m = 1.493 \qquad \mu = 0.8927 \qquad \tau = 331.8$$

$$f = 0.8808 \qquad \gamma = 9.516 \qquad \pi_{pl}^{\max} = 0.09957$$

which correspond to the engine parameters listed at the bottom of Table 2.

The appearance of an extremum in π_{pl}^{max} extends through the other interesting system parameters and may be traced easily in the numerical results of Table 2. Figures 5 and 6 show clearly the presence of a minimum in the containment factor at $f = 0.85$ and a maximum in the total fuel lost W_F^{ex} at $f = 0.87$. These may also be derived analytically, but not easily. Considerable algebraic complication arises here because in determining the average fuel concentration in the reactor, use must be made of the transcendental criticality relation.

VI. CONCLUSIONS

Some general observations and conclusions can be drawn from the results of this analysis.

(1) The primary constraint in determining containment requirements is the total fuel loss allowed during the propulsion period.

(2) Given a mission, the containment requirements can be directly related to the engine parameters and the allowable fuel lost per propulsion period.

(3) In the high-thrust application, gaseous-fuel nuclear rockets offer significant improvement over conventional (temperature-limited) direct nuclear systems when the containment factor exceeds about 50.

(4) Similar containment requirements apply to the low-thrust systems if the nuclear fuel loss is to be kept within reasonable bounds and/or the system performance is to be comparable to nuclear-electric systems for near-Earth planet missions (Ref. 5).

(5) Containment factors appreciably less than 50 yield fairly large specific-impulse ratios for the low-thrust systems, and would be attractive if the attendant loss of nuclear material were acceptable.

REFERENCES

1. Meghreblian, R. V., "Gaseous Propulsion Reactors," *Nucleonics,* Vol. 19, 1961, pp. 95–99.

2. Kerrebrock, J. L., and R. V. Meghreblian, "Vortex Containment for the Gaseous-Fission Rocket," *Journal of the Aerospace Sciences,* Vol. 28, 1961, pp. 710–724.

3. Nelson, S. T., *The Plasma Core Reactor,* GM 60-7630.2-9, Space Technology Laboratories, Inc., Los Angeles, Calif., June 22, 1960.

4. Weinstein, H., and R. G. Ragsdale, *A Coaxial Flow Reactor—a Gaseous Reactor Concept,* ARS Preprint 1518-60, presented at ARS 15th Annual Meeting, Washington, D. C., December 5–8, 1960.

5. Meghreblian, R. V., "Prospects for Advanced Nuclear Systems," *Astronautica Acta,* Vol. 7, 1961, pp. 276–289.

6. Meghreblian, R. V., "Gaseous Fission Reactors for Booster Propulsion," *ARS Journal,* Vol. 32, 1962, pp. 13–21.

REFERENCES (Cont'd)

7. Altman, D., *Thermodynamic Properties and Calculated Rocket Performance of Hydrogen to 20,000°K*, Report No. 20-106, Jet Propulsion Laboratory, Pasadena, Calif., September 3, 1956.

8. Meghreblian, R. V., *Gaseous Fission Reactors for Spacecraft Propulsion*, Technical Report No. 32-42, Jet Propulsion Laboratory, Pasadena, Calif., July 6, 1960.

9. Meghreblian, R. V., *Thermal Radiation in Gaseous Fission Reactors for Propulsion*, Technical Report No. 32-139, Pasadena, Calif., July 24, 1961.

ACKNOWLEDGMENT

The author thanks Dr. H. Stumpf for reviewing and checking the analysis, and Mr. T. Thomas for programming the computations.